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HEAT TRANSFER AND FLOW IN A SHALLOW RECTANGULAR CAVITY WITH SUBSONIC TURBULENT AIR FLOW

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Abstract—Pressure coefficients, heat-transfer coefficients, and recovery factors have been measured on the surface of shallow cavities of rectangular form, with length to depth ratios of 2 to 5, for air flow over the cavity at speeds from 160 to 590 ft/s. The flow over the surfaces of the cavity is indicated to be two dimensional over a fraction of the cavity surface and, for this region, partial rationalization of the heat-transfer coefficient is achieved from the indication of temperature and mean speed profiles obtained at various points downstream of separation. The importance of the free shear layer across the top of the cavity is emphasized and a number of its features are shown to correspond to those of the free jet boundary.

NOMENCLATURE

- b, width of shear layer;
- c, heat capacity;
- c_x, local friction coefficient;
- c_p, pressure coefficient;
- h, heat-transfer coefficient;
- H, depth of the cavity;
- k, thermal conductivity;
- L, length of the cavity;
- p, static pressure;
- q, heat flux;
- Q_R, volume rate of reverse flow;
- T, state temperature, T_a adiabatic wall temperature, T_W wall temperature;
- T_t, stagnation (total) temperature;
- u, velocity in x direction;
- u_R, maximum value of reverse velocity;
- W, the group $(h/k(\mu_0/\rho_s u_s)^{0.8}$;
- x, distance in direction of flow or distance along the wall;
- y, distance normal to the bottom surface (origin varies in some situations);
- α, thermal diffusivity;
- μ, viscosity;
- ρ, density;
- τ_W, shear at the wall.

Subscripts

- 0, free stream stagnation temperature;
- 1, free stream;
- 2, inner edge of shear layer;
- s, just upstream of the front face.

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INTRODUCTION

A PARTICULAR KIND of separated flow exists in a rectangular cavity situated in an otherwise uniform surface. If the cavity is not too long in relation to its depth, the free shear layer, initiated at the separation which occurs at the upstream edge of the cavity, spans the cavity completely and impinges partially on the back face of the cavity. There it is in part diverted into the cavity to provide the back flow in the cavity which ultimately returns to the free shear layer to provide the traction that produces the development of that layer. While flows of a similar nature would occur in cavities of other shape, the rectangular form is attractive for heat-transfer studies because of the relative ease of providing for the heating of the surface and associated instrumentation and, most important, the adjustment of the cavity length. It is much for these reasons that existing results for the flow and heat transfer in cavities deal with the rectangular form and of these investigations there are typical those of Charwat *et al.* [1] for the heat transfer and flow, of Tani and Komoda [2] for parts of the flow, and of Seban and Fox [3] for a rectangular cavity formed by a spanwise step placed downstream of a downward step in the surface of a plate.

It was the aim of these experimental investigations, and it was also the object of the experimental investigation reported here, to define more precisely the nature of the flow in the separated region and to relate to that flow the observed rates of heat transfer, and in this investigation the observed adiabatic wall temperatures as well. The nature of the results that are presented indicates that an overall rationalization is still remote but that certain elements of the flow and heat transfer are explicable, particularly in respect to the influence of the free shear layer on the internal flow and heat transfer.

The system considered here was a rectangular notch, 2·05 inches deep, with lengths between approximately 4 and 10 in, produced by alteration of the downstream position of the back face. The notch was contained in the model illustrated in Fig. 1, which shows its position in the wind tunnel test section; this section is 6 in wide and the model spanned the section completely. The air flow just upstream of the notch was, at 160 ft/s, laminar with a thickness of 0·005 in. Results were obtained for velocities between 160 and 590 ft/s.

In the initial version of the model, pressure taps were located, primarily on the centerline, on the front section, on the faces and floor of the notch and on the surface downstream of the notch. The pressures that were measured are presented as pressure coefficients, in terms of the dynamic head just upstream of the notch, and the static pressure there:

$$c_p = (p - p_s) \Big/ \frac{\rho_s u_s^2}{2}. \tag{1}$$

At the time these measurements were made the notch was extended only to 7 inches in length, thus the pressure measurements within the notch are limited, and partial results for the longer notches were limited to the indication obtained from a static tube placed at the wall.

In the second version of the model the internal surfaces of the notch were replaced by surfaces containing nichrome ribbons, 0·002 in thick and 0·50 in wide, placed spanwise, with 0·005 in spaces between them. Electric current passing through these ribbons in series provided for constant heat dissipation over the surfaces of the notch. Thermocouples were placed in the bakelite immediately below the ribbons, with most thermocouples located on the centerline. With unheated operation these thermocouples indicated the adiabatic wall temperature, from which the recovery factors were determined, as:

$$r = 1 - \frac{T_a - T_{ts}}{u_s^2/2c}. \tag{2}$$

With heated operation the heat-transfer coefficients were obtained directly from the heat flux inferred from the electrical dissipation, together with the wall temperature and the associated adiabatic wall temperature,

$$h = \frac{q}{T_W - T_a}. \tag{3}$$

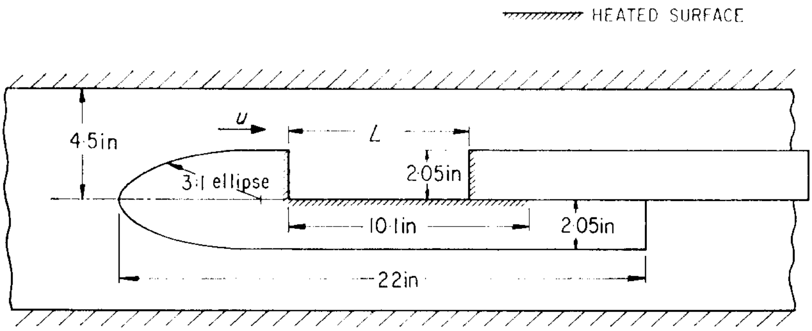

FIG. 1. The cavity model.
The length L was adjustable in 0·505 in increments up to L = 10·10 in.

Thermocouples located on the internal surface of the bakelite indicated that the heat losses into the bakelite were small, as are those externally by radiation. These errors, combined with the effect of longitudinal conduction, are the largest at the lowest speeds and for them the heat rate as used in equation (2) may be five per cent high. With heating, the maximum difference between the wall and free stream stagnation temperatures was of the order of 20 degF; stagnation temperatures were of the order of 85°F.

Of the results that have been obtained from this model, those for cavity lengths of 3·75 in and less have already been reported by Fox [4, 5] and this paper presents the results for cavity lengths from 4 to 10 in.

Values of the pressure coefficient, heat-transfer coefficient and recovery factor, were measured for notches approximately 4, 5, 6, 7, 8 and 10 inches in length. The 10 in length is close to the maximum that can be attained without the point of reattachment shifting to the floor of the cavity, while the 4 in length, $L/H \approx 2$, is close to the size in which Maull and East [6] found a three-dimensional motion associated with multiple vortices which existed at the front of the cavity. While they implied that such motions might not exist for $L/H > 2$, it is shown later that three-dimensional motion does exist in the forward part of all of the notches in the range of sizes considered here.

Tani and Komoda [2] have also suggested that the range of cavity sizes considered here constitutes a separate regime of flows, which they estimated to begin near $L/H = 1{\cdot}70$, with the longer "shallow" notches distinguished by the location of the maximum pressure coefficient below the top edge of the back face. This is not apparent in the present results but in them the pressure coefficient on the back face is positive throughout, and this essentially is evident also in Tani's and Komoda's results. With the present system there is a radical alteration in the pattern of the pressure coefficient for the 3 in ($L/H = 1{\cdot}5$) notch, and the change from "shallow" to "deep" notch behavior occurs in this region.

RESULTS

The nature of the results that have been obtained is demonstrated for the 4·04 in, 7·07 in, and 10·10 in notches on Figs. 2–4, on which the results are presented in terms of an abscissa which represents distance along the front, bottom, and back surfaces of the rectangular cavity. The free stream edges of the front and back faces are the extremes of the abscissa, and heavier grid lines indicate the internal corners of the notch.

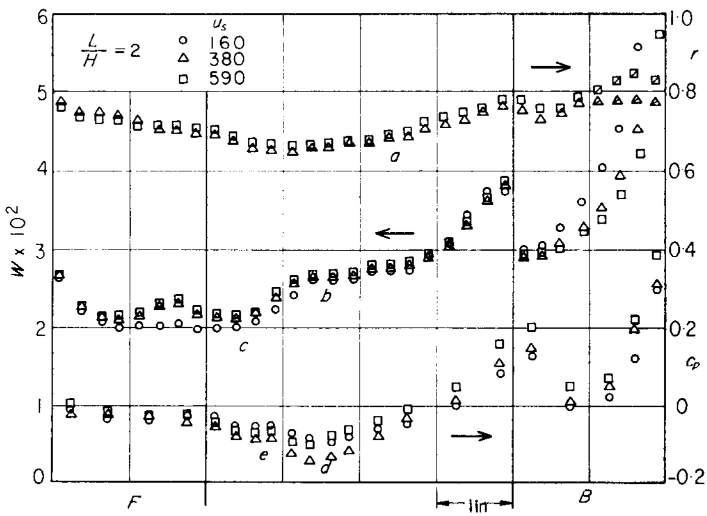

FIG. 2. Results for the surface of the 4·04 in cavity.
The abscissa contains the front face, to the left; the back face to the right; and the bottom surface placed between these faces. The letters refer to Table 1.

On the figures the pressure coefficients and recovery factors are direct evaluations of the experimental results, while the heat-transfer coefficients have been incorporated into the group W, which is

$$\frac{h}{k_0}\left(\frac{\mu_0}{\rho_s u_s}\right)^{0\cdot 8}.$$

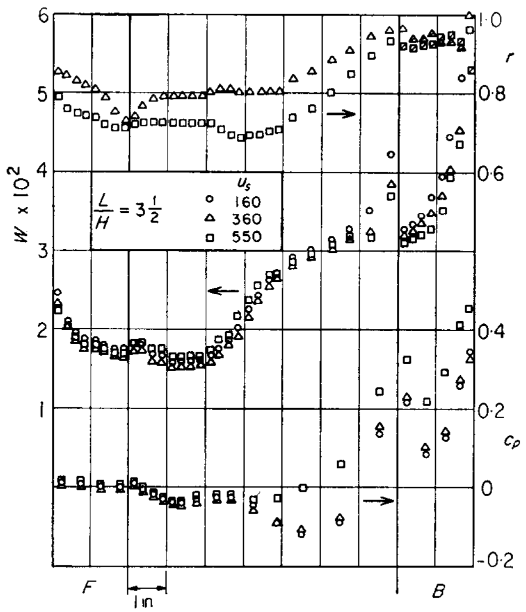

FIG. 3. Results for the surface of the 7·07 in cavity.

The choice of this group is somewhat arbitrary, though its selection was obviously guided by the form of correlation associated with turbulent flow over a flat plate. The basis of property evaluation, with the thermal conductivity and the viscosity evaluated at the stagnation temperature (essentially the same as the surface temperature), and the density at the temperature and pressure at the step, is one which serves to correlate fairly the present experimental results, though of itself it cannot be regarded as fundamental. Other bases could be selected; if the density is taken at the stagnation condition instead of at the step, a satisfactory degree of correlation can be achieved if the exponent of 0·8 is altered to 0·7.

The results of the surface measurements as shown on Figs. 2–4 are intended primarily as a general view of the heat-transfer performance of notches in this size range. The details of the flow and heat transfer are discussed in ensuing sections, but specific attention may be drawn to common features of the results that are shown on the figures. Of these, a significant similarity exists in the heat-transfer coefficient, which, from its maximum at the top of the back face, diminishes toward the front, a variation that appears to be connected with the position at which the pressure coefficient is at a minimum. This minimum moves toward the back of the

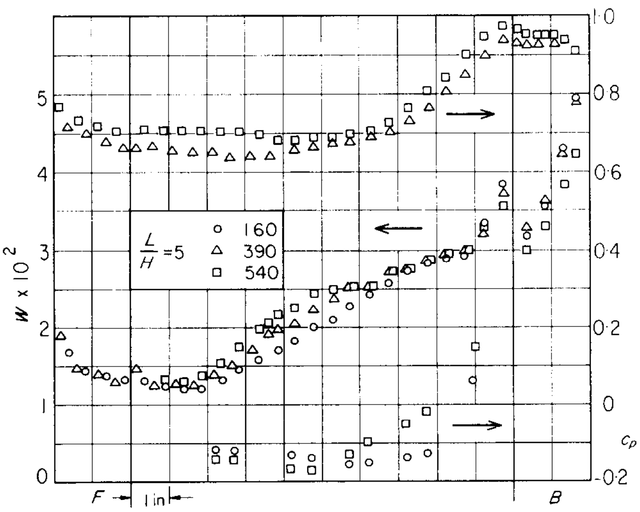

FIG. 4. Results for the surface of the 10·10 in cavity.†

† Note added in proof: on Fig. 4 the ordinate for c_p should span $-0{\cdot}1$ to $+0{\cdot}1$ rather than $-0{\cdot}2$ to $+0{\cdot}2$ as indicated.

notch as the length of the notch increases. The variation of the recovery factor is also altered in this region, and, in the longer notches it is there that the recovery factors at different velocities are different. It is noted later that it is in the region near the end of the pressure rise from its minimum, that separation of the forward flow occurs, with a region of three-dimensional flow between this point and the front face of the notch.

The order of the observed heat-transfer coefficient can be appraised in terms of the average value of the group

$$\frac{h}{k}\left(\frac{\nu}{u_1}\right)^{0\cdot 8}$$

indicated by the Colburn equation for turbulent flow over an isothermal plate of the same length as the notch:

$$\left[\frac{h}{k}\left(\frac{\nu}{u_1}\right)^{0\cdot 8}\right]_{\text{avg}} = \frac{0\cdot 034}{L^{0\cdot 2}}. \tag{4}$$

For the 4, 7 and 10 in notches, for which the results are shown on Figs. 2–4, the values of W_{avg} obtained from equation (4) are 0·041, 0·038 and 0·035. These are far above the average of the local coefficients on the bottom of the notches. If a comparison of total heat transfer is to be made, however, the additional contribution of the front and back surfaces must be considered also. Their influence depends on the relative length of the notches and, using the results for the lowest speed run, the ratio of the total heat transfer from the notch to that from a flat plate having a length equal to that of the notch is 1·22, 1·05 and 0·95 for the 4, 7 and 10 in notches, respectively. These ratios indicate partially the increased contribution of the faces of the notch when the length-to-depth ratio is small. These ratios are also too small, because in the forward region the values of the heat-transfer coefficient on the centerline are, particularly on the front face, lower than the coefficients on the remainder of the surface.

The pressure coefficients in each of the notches are essentially the same for the two lower speeds, while for the highest speed there is an increase in the coefficient on the back face and on the rear part of the bottom face. While this may well be connected with an effect of compressibility it apparently cannot be explained on this basis and the difference in the coefficients for the highest speed might be construed to indicate an alteration of the flow in the cavity. The results for the heat transfer on the back face for the high speed operation do show some departure from a correlation with free stream velocity but the departure is scarcely large enough to sustain an argument for any pronounced change in the nature of the flow.

To examine the results in more detail, there are next considered some observations concerning the flow, and these are followed by more detailed comments regarding the nature of the heat-transfer coefficient and the adiabatic wall temperatures.

FLOW

The shear layer

The separation which occurs at the front face of the notch produces a free shear layer between the external stream and the region of lower velocity within the cavity. This internal flow is produced by the deflection into the cavity of part of the shear layer as it impinges on the back face and the consideration of the flow in the cavity therefore begins appropriately with an examination of the shear layer itself. While there is no completely adequate analytical solution for this layer, it is at least partially described by an approximate integral solution in which the velocity profile is taken as one which is a reasonable approximation for the free jet boundary.

Abramovich [7] has presented such a solution, in terms of the flow downstream of a backward facing step, in which it is assumed that the free stream velocity is constant, so that the streamlines of the outer flow are parallel to an x axis, which originates at the separation point and is parallel to the plane surface downstream of the step. He assumes a velocity profile of the form

$$\frac{u}{u_1} = 1 - \left(1 - \frac{u_2}{u_1}\right) f(\eta)$$

$$\text{with} \quad f(\eta) = \left((1 - \eta^{3/2})^2 \quad \text{and} \quad \eta = \frac{y - y_2}{y_1 - y_2}\right). \tag{5}$$

In addition it is assumed that the shear layer grows linearly according to the law

$$y_1 - y_2 = b = 0{\cdot}30x. \qquad (6)$$

These assumptions together with the equations of continuity and momentum, orient the shear layer by specifying y_1 as a function of b, and specify the velocity u_2, which is uniform between the point y_2 and the lower boundary of the flow. This velocity u_2 is negative and inherently this provides for the forward flow, part of which continually re-enters the shear layer to provide the traction which causes the shear layer to broaden. Question arises immediately however, as to the tenability of the assumed velocity profile, which has been shown to be realistic only for situations in which u_2 is either zero or positive.

Figure 6 shows some of the lines of constant velocity ratio, u/u_1, obtained from the analysis of Abramovich and shows also one streamline, the zero streamline. These are shown to a distance of $x/H = 5$; Abramovich suggested $x/H = 4{\cdot}7$ as a logical termination of this analysis and from other considerations he suggested $x/H = 6$ as the reattachment point for the system of the backward facing step.

Values of the mean speed were deduced from impact and static tube surveys made on the centerline, with the axes of the 0·050 in diameter tubes always parallel to the bottom of the notch, with the tubes reversed near the bottom of the notch to measure the reverse velocity there. Therefore some errors in indication are present due to such inclination of the flow as exists. Profiles were obtained at various locations in the 4, 7, 8 and 10 in notches, and a typical result is shown on Fig. 5 for position 5·75 in downstream in the 8 in notch. There is shown also the Abramovich solution for this position and it is clear that this is a good approximation to the actual profile for velocity ratios, u/u_1, greater than 0·4; for smaller velocity ratios the values diverge. In the region $0{\cdot}7 < y < 1{\cdot}2$ in no consistent results were obtained, the impact pressures tending to be less than static regardless of tube direction. Moreover, the values of velocity to either side of this central region may be in error, and all this uncertainty obviates the possibility of specifying the dividing streamline

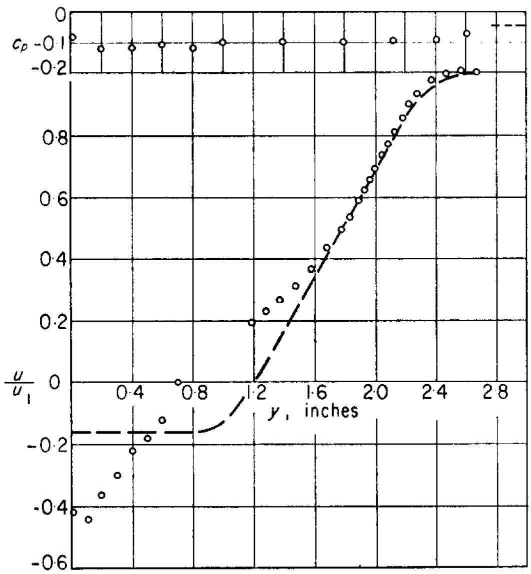

FIG. 5. Velocity profile near 6 in from the front face in the 8·08 in notch, at low speed.

The curve is the indication of the Abramovich analysis. Pressure coefficient at this station are shown at the top of the figure, where the dashed line indicates the final values at greater distances from the wall.

by an integration beginning at the bottom of the cavity.

An alternate but less exact method of specifying the dividing streamline consists of assuming a free stream velocity which is, as confirmed experimentally, invariable in the normal direction. Then continuity can be applied, integrating downward from the top of the flow cross-section. In a sense, this neglects the influence of the boundary layers on the tunnel walls, but these walls diverge slightly to compensate for these layers and it is known that in the absence of a model, the static pressure is essentially uniform in the flow direction. This supports the application of the continuity equation in this direction, using the local free stream velocoty. For the profile indicated on Fig. 5, there is found a location near $y = 2{\cdot}05$ for the dividing streamline; remarkably, this is the position of the top surface of the cavity. Reference to Fig. 6 indicates this to be above the position indicated by the Abramovich solution.

Figure 6 shows points for constant velocity ratios, u/u_1, obtained from other velocity distributions in the outer region of the shear layer, where $u/u_1 > 0{\cdot}40$, and for the 7, 8, and 10 in cavities these points indicate the same kind of agreement with the Abramovich profile that is shown for the particular traverse of Fig. 5. While all these results were obtained at a free stream velocity of approximately 160 ft/s, there are indicated at two positions on the figure, by "plus" symbols, corresponding results obtained at a free stream velocity of approximately 450 ft/s; the agreement is as good. For all of these profiles, the location of the zero streamline by the method indicated above gave location near $y = 2{\cdot}05$ in (this is $y = 0$ on Fig. 6 where these distances are measured with reference to the top of the front face, the separation point).

A separate representation is used on Fig. 6 for three profiles obtained in the 4 in notch. With this notch the distances are short, the shear layer is narrow, and the positions of given velocity ratios are subject to greater error. It appears, nevertheless, that the correspondence to the Abramovich analysis is poorer. At the two downstream stations, moreover, the zero streamline is indicated to be nearer $y = 2{\cdot}12$ in (0·07 in on the scale of the figure). This suggests that the shear layer may rise at first and then be depressed downstream, nearer the step, where the impact tube could not be inserted.

Figure 6 shows also the free stream pressure coefficients. These are close to, but generally a little higher than, the corresponding pressures at the bottom of the notch. The pressures within the notch are a little lower, as indicated by the representation of the local pressure coefficient that is shown on Fig. 5, a representation that may be regarded as typical, except near the back face. There the pressure coefficient in the free stream remains negative, while that at the bottom of the notch is positive. The positive value is typical of the internal region and the variation to the lower external pressure takes place in the outer part of the shear layer.

Back face

The only evidence concerning the flow in the reattachment region is that of the pressure coefficients measured on the back face of the notch and with only five pressure taps available on the back face a fine structure in the distribution of the pressure distribution may be overlooked. For instance, Tani and Komoda [2] indicated a maximum located at a distance below the top edge of the back face and it is possible that such a maximum could have existed between the first two taps in the present model;

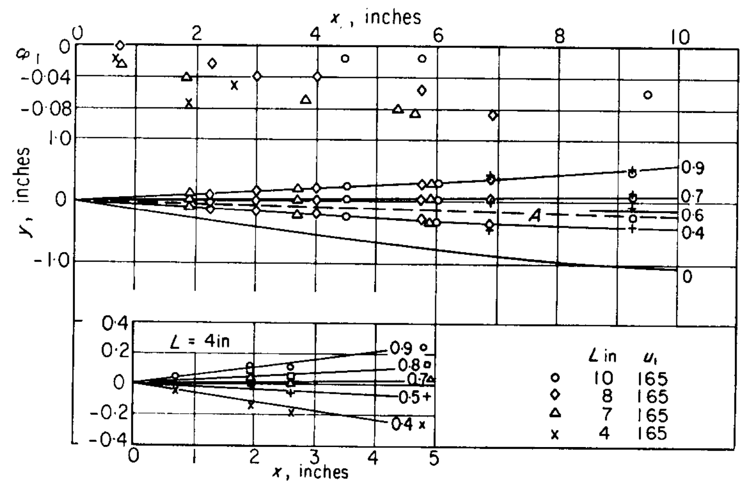

FIG. 6. Constant velocity ratios in the free shear layer.

The lines of constant velocity ratio are from the Abramovich theory and Curve A is the predicted location of the zero streamline from that theory. The results for the 4 in notch are shown separately with an expanded ordinate. Free stream pressure coefficients are shown at the top of the figure.

otherwise, however, the distribution of pressure is typical of that of Tani and Komoda. For this range of length-to-depth ratios the pressure coefficients are all positive and they increase as the length-to-height ratio increases. This is exhibited in Fig. 7 by the results for the low velocity of 165 ft/s; there all coefficients tend to increase as the notch length increases, except for an ambiguous behavior at the tap nearest the top edge.

Figure 7 also contains values of the pressure coefficient on the surface downstream of the notch. The decreased pressure coefficients there indicate the separation in this region, a separation which is apparently completed by reattachment before the station located one inch downstream of the top of the back face. Velocity profiles obtained at that station support the estimate of reattachment slightly upstream of that point.

The nature of the pressure variation within the fluid in the reattachment region not being known, any attempt at relation of the observed pressure coefficients to the velocity distribution in the shear layer must be tenuous at best. In view of this, the often adopted view of preservation of the dynamic head in the shear layer may be considered. Disregarding the point for the 5 in notch, the pressure coefficient, at 165 ft/s, near the top edge of the back face ranges from 0·33 to 0·36. In terms of an assumed pressure equal to static pressure at the step to exist upstream of the notch, these coefficients imply velocity ratios of 0·58 to 0·60, or, in terms of a free stream pressure coefficient of 0·05 just upstream of the back face, they imply velocity ratios of 0·62 to 0·64. While these are magnitudes which might be expected in terms of the lines of constant velocity ratio that are shown on Fig. 6, the information obtained from the pressure coefficients on the wall is insufficient to specify the location of the dividing streamline on the back face.

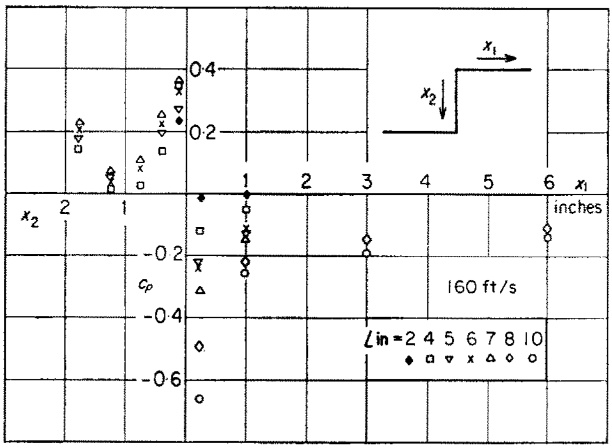

FIG. 7. Pressure coefficients on the back face and downstream of the back face, at low speed.

It would appear that it is above the location that might be deduced from the Abramovich analysis and below the value of $y = 2{\cdot}05$ indicated from the experimental velocity profiles.

While the pressure near the top of the back face is of the order of what can be deduced from the velocity profiles in the shear layer, the rest of the pressure distribution on the wall cannot be estimated in this way. A deduction based on the dynamic head of the approaching flow gives a pressure coefficient which diminishes far more rapidly with distance downward from the top corner than does the experimental distribution. On the bottom portion of the back face the pressures again rise, due in part to the deceleration of the flow that is about to turn the bottom corner; no adequate means are at hand for predicting the pressure rise in this region.

Bottom surface

The pressure coefficients in the back corner indicate a relative symmetry that is expected from the turning of the flow that occurs there. Going upstream from this point the pressure coefficient becomes negative; these negative values may be associated with the negative pressure coefficients in the free stream that are shown on Fig. 6 but the values at the wall are slightly lower. Further forward the pressure coefficients rise to relatively constant values which persevere to the inner front corner.

The direction of the flow near the wall of the cavity was observed in some of the notches by means of string tufts attached to the walls and the orientation of these tufts revealed an essentially uniform downward flow on the back face and a forward flow on the bottom, up to a region definable within about an inch in length. There the flow appeared to be oscillatory, with an indication of both forward and backward, as well as some spanwise velocity. This region is considered as one of separation of the forward flow from the bottom of the notch. Between this region and the front face the tufts indicated an oscillatory three-dimensional motion, with a tendency, on the average, to a downstream direction. The direction of motion there cannot be specified firmly because of the limited number of tuft locations and the inconsistency of direction of the tufts at the various locations.

On the front face the indication of the tufts was of inward flow from the sides to the centerline; only near the centerline was there an indication of upward flow.

This general picture was obtained for the 4, 7, 8 and 10 in notches. As a check, the 2 in ($L/H = 1$) notch was also examined, with the finding of uniform forward flow over all faces of the notch, there being no indication of oscillatory behavior, separation, or any spanwise velocities.

There is an apparent relation between the nature of the pressure coefficient, the heat-transfer coefficient, the recovery factor, and the location of the separation region in the bottom of the notch. No precise definition can be given because of the relatively gradual changes in the behavior of the quantities concerned, but Fig. 2 indicates by letters the regions in which alteration of their behavior occurs: *a*, an end to the decrease in the recovery factor; *b*, the beginning of a more rapid decrease in the heat-transfer coefficient; *c*, the termination of that decrease; *d*, the minimum in the pressure coefficient; *e*, the end of the pressure rise, going forward from *d*. Table 1 indicates these regions for various notch sizes and includes also the value, *S*, where "separation", or the end of the definite forward flow region, exists. These are specified in terms of the distance forward from the back face. There is particular correspondence between the separation region and *e*, the end of the pressure rise, and *c*, the end of the decrease in the heat-transfer coefficient.

In the region of forward flow the form of the velocity profile near the wall is at first rather the reflection of that part of the downstream flow in the shear layer as may be considered to be diverted into the cavity. This is indicated by Fig. 5 for a position 2 in forward of the back face. There, too, the retardation of the flow near the wall is evident. At forward positions, the profile of the reverse flow tends to flatten and the maximum velocity, u_W, occurs at greater distances from the wall. The profiles obtained near the wall were integrated to evaluate the forward flow, $Q_R = \int_0 u \, dy$ and the values obtained in this way are shown as $Q_R/u_s H$ on Fig. 8. The

Table 1. Distances forward from back face, in

L	a	b	c	d	e	S
4	2	2·5	3·5	2·5	3	3·5
5	2	2	4	2·5	3·5	
6	2·5	2·5	4	2	3·5	
7	3·5	4·5	4·5	2·5	4	4
8	4	4	5·5	2·5	4	5
10	5	6	7·5	(4)	(7)	7·5

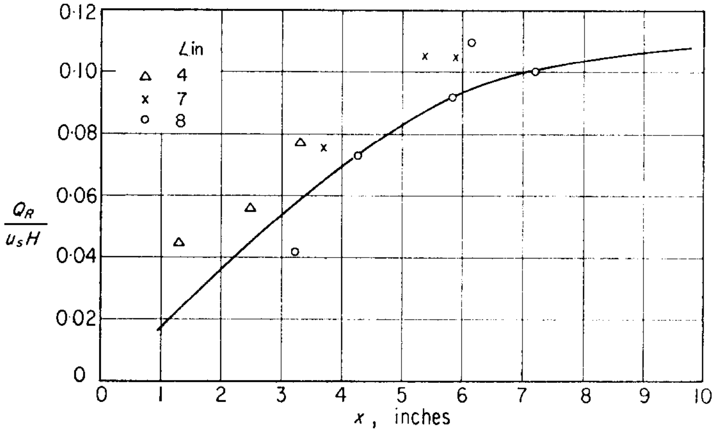

FIG. 8. Volume rate of the reverse flow.

The curve is that given by the Abramovich analysis. Distance is measured downstream from the front face.

prediction of the Abramovich theory is shown there also and, surprisingly, there is correspondence with the measurements for the 8 in notch, despite the difference between the actual velocity profile near the wall and the uniform velocity postulated by the theory. The experimental values for the 4 and 7 in notches are higher, but they are related in form to the prediction from the theory.

Figure 9(a) shows the reverse velocities predicted by the theory and, as indicated on Fig. 5, these are much lower than the maximum values of the reverse velocity, which are shown also on Fig. 9(a). These experimental values are not consistent in terms of the distance downstream from the front face and Fig. 9(b) shows that the values for the three notch sizes are at least organized better by a representation in terms of relative position in the notch, a finding noted before by Seban and Fox [3].

In the 8 in notch a small impact tube was used to determine the velocity distribution very near to the wall at positions 1 and 4 in forward of the back face. Attempts at orienting the measured velocities to the law of the wall, with a von Kármán sublayer region, proved to be relatively indecisive, as was found by Fox [4] in respect to similar measurements in shorter notches. In the two positions in the 8 in notch with a free stream velocity of 160 ft/s, this kind of appraisal led to estimates of friction velocities,

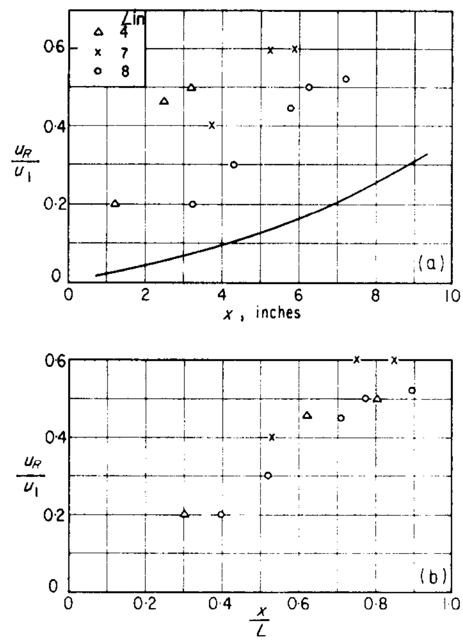

FIG. 9. Reverse flow velocities.

Figure (a) shows the maximum velocity of the reverse flow in terms of distance downstream from the front face. Figure (b) shows the same results in terms of the ratio of that distance to the cavity length.

$\sqrt{(\tau_W/\rho)}$ of 6 and 3·8 ft/s respectively. In comparison, it can be noted that a turbulent boundary layer on a plate, with a free stream velocity of 80 ft/s, would be expected to produce friction velocities of 4·8 and 4·15 at positions of 1 and 4 in from the leading edge. The velocity of 80 ft/s is typical of that which exists near the wall of the cavity for a free stream velocity of 160 ft/s and the correspondence with the magnitude of the uncertain measurements is significant in relation to subsequent considerations of the heat transfer on the bottom surface.

HEAT TRANSFER

Heat is transferred from the wall to the adjacent fluid and then to the shear layer as this fluid returns to it, though the eddy diffusivity in the cavity is so high that some direct transfer could possibly occur. The nature of the shear layer itself is such that the total heat transfer from the wall can be contained in it without requiring, at its inner edge, a temperature that is much in excess of the free stream temperature. If a constant temperature is assumed at the inner edge, this temperature can be determined analytically as that which is necessary to provide at the downstream end of the cavity, the location on the outside of the dividing streamline of all of the energy transferred from the wall of the cavity. If dissipation is neglected for convenience, and if the Prandtl number is taken to be unity, together with equal diffusivities for the transfers of heat and momentum, then the state temperature is a linear function of velocity,

$$\frac{T - T_1}{T_2 - T_1} = \frac{u - u_1}{u_2 - u_1} \tag{7}$$

and, for the energy to be contained outside of the dividing streamline, at $y = y_0$

$$\frac{Q_W}{\rho c} = \int_{y_0}^{\infty} (T - T_1)\, u \,\mathrm{d}y. \tag{8}$$

This can be evaluated once the velocity distribution is specified and it is found that the temperature difference $(T_2 - T_1)$ is small compared to the observed differences between wall and free stream temperatures. Korst [8] has made a number of evaluations of this type.

Temperature traverses made in the 8 in notch, at positions of 4 and 7 in from the front face, confirmed the relatively small contribution of the shear layer to the total heat-transfer resistance. The temperature difference, $(T_2 - T_1)$, was indicated to be about ten per cent of the total temperature difference, $(T_W - T_1)$, at both of the locations and consequently, with temperature differences, $(T_W - T_1)$, of 10 degF and 15 degF, the temperature profile across the mixing zone was difficult to measure precisely. On the cavity side of the shear layer the temperature was uniform, and the remaining ninety per cent of the temperature rise occurred very close to the wall, within 0·03 in at the 7 in station and within 0·10 in at the 4 in station. Because of the small contribution of the shear layer, the ensuing discussion of the heat-transfer coefficients retains the original specification of the heat-transfer coefficients, equation (3), though it is to be noted that, in terms of temperatures of the fluid close to the wall, these coefficients, and consequently the group W, would be about ten per cent larger.

Back surface

The highest heat-transfer coefficients occur at the top of the back face, in the region of reattachment; they decrease with distance toward the bottom and they indicate no particular effect due to the pressure rise that occurs near the bottom of the back face. These coefficients, contained in the group W, already indicated on Figs. 2–4, are shown again on Fig. 10 for the lowest and highest speeds. For clarity, results are given only for the 4, 6 and 10 in notches, though the results for the other notches would be positioned in the same band of points. For the lower speed there is near the top of the back face a small trend with notch size, but except for the upper position, there is less than ten per cent change in the heat-transfer coefficient and the points are approximated by a straight line, indicated as a on the figure, having a slope of $-0{\cdot}30$. It is judged that, despite the greater amount of fluid diverted into the longer notches, the velocity and its distribution along the wall must remain essentially the same.

There is an effect of speed, shown on Fig. 10 by the results for the highest speed, to indicate the failure of the correlation group W to account completely for the effect of air speed, as is also apparent on Figs. 2–4. The position of line a illustrates, however, that the discrepancy

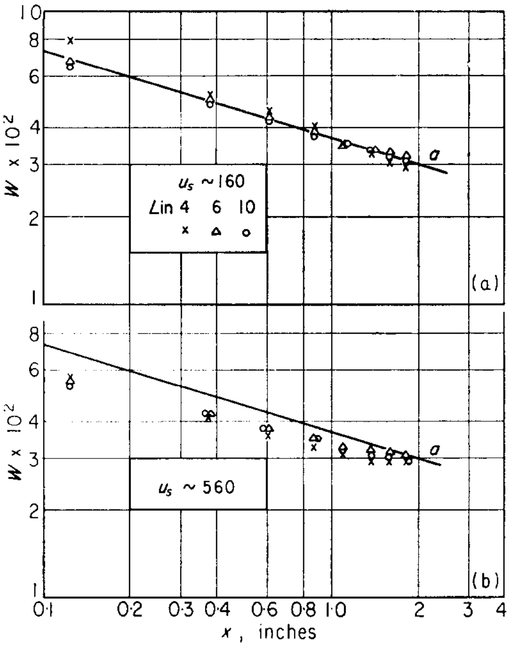

FIG. 10. Heat-transfer coefficients on the back face. The distance x is measured from the free stream edge of the face. Line a indicates the mean value for the low speed.

varies with position, so that a lower exponent for the factor

$$\left(\frac{\mu_0}{\rho_s u_s}\right)^{0 \cdot 8}$$

would improve the velocity correlation near the top of the back face but make it inferior near the bottom of the face.

There is some similarity of the situation in the reattachment region to the situation of a two-dimensional jet impinging normally on a plate, which was the subject of investigation by Schauer and Eustis [9] and for which they found in the stagnation region a dependence on distance like $h \sim x^{-0 \cdot 2}$. A comparison in magnitude is possible if similar velocities are taken for the jet near the plate, and on this basis the results of Schauer and Eustis give a heat-transfer coefficient that is twice as high as those measured near the top of the back face. While the approaching velocity profile is different in the notch system than it is is in the free jet, a somewhat better correspondence would be anticipated.

Bottom surface

As the flow turns the corner from the back face to the bottom there is an increase in the local heat-transfer coefficient, which suggests some degree of temperature equalization of the flow, possibly due to a separation and a small eddy very close to the corner. This suggests a representation of the heat-transfer coefficient on the bottom surface in terms of distance from that corner and Fig. 11 pictures the results for all of

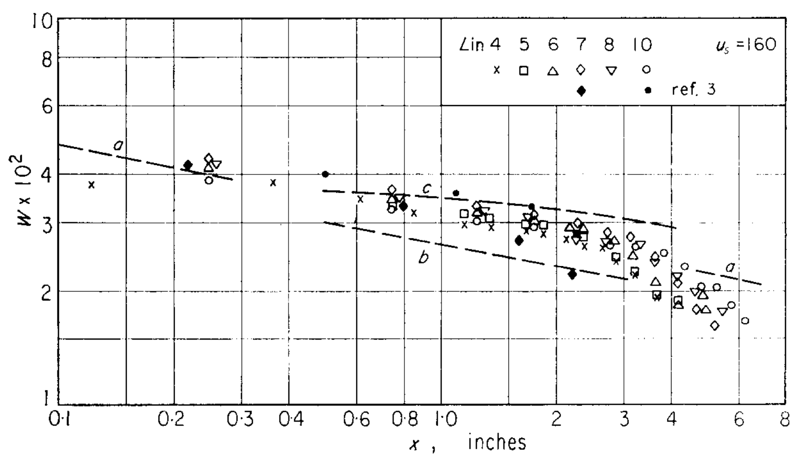

FIG. 11. Heat transfer on the bottom surface.
The distance is measured forward from the back face. All results refer to a free stream velocity of about 160 ft/s. Line a indicates the mean value where a power law applies. Lines b and c are from equations (9) and (10).

the notches in this way, using only the results for the lowest speed, about 160 ft/s. In the initial distance from the corner there is an approximation to power law behavior, illustrated by line *a*, though there are systematic deviations from a mean which do depend on notch length. Departure from this behavior occurs first for the shortest notch and subsequently for the longer notches. This is in the region *b* defined in Table 1, though the departure for the 10 in notch occurs a little earlier than indicated by that estimate. The subsequent final alteration in the coefficient, at or to a minimum value, is likewise in general agreement with point *c* and the "separation" point.

Figure 11 also contains points from the system used by Seban and Fox [3], in which a rectangular cavity was formed by a fence placed downstream of a backward facing step, in which the cavity depth was 0·81 in. At short distances these points are within ten per cent of the mean of the present results but they depart at different positions than do the present results. This, together with the observation of the position of the present results and the apparent relation of the distribution of the maximum velocity to the relative cavity length that is indicated by Fig. 9(b), suggests the representation of Fig. 12 where the abscissa is the ratio of the distance from the back face to the cavity length. This does not materially improve the correlation of the present results but it does present other aspects of the situation. It groups more successfully the results for the cavities from 4 to 8 inches in length but displaces the results for the 10 in cavity. It reverses the order of the departure, at point *b*, from the relatively linear distribution and indicates that for cavities between 5 and 8 inches in length this point is at almost a fixed fraction of the cavity length. The results for the cavities from the system of reference 3 are now in mutual agreement, but are about ten per cent above the mean of the present values, with departure from linear behavior occurring in the same relative region. This region of departure is, however, nearer to the front face than for the cavities of the present investigation.

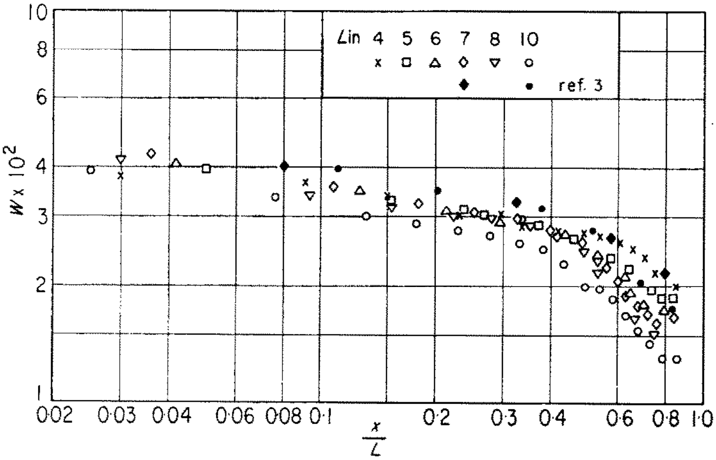

FIG. 12. Heat transfer on the bottom surface.
The same results as on Fig. 11, with the abscissa the ratio of the distance from the back face to the length the cavity.

While the representation of Fig. 12 indicates the importance of the actual variation of the reverse velocity along the length of the cavity, comparison of the results to those of more conventional systems is made more readily on the basis of Fig. 11 in which the dependence is directly on distance and for which appraisal can be made in terms of the heat transfer produced by turbulent flow on a flat plate. For the flat plate on which the velocity is u_R the von Kármán analogy gives, for a Prandtl number of 0·70,

$$\frac{h}{\rho c u_R} = \frac{c_f}{2} \bigg/ \left(1 - 3\sqrt{\frac{c_f}{2}}\right) \tag{9}$$

or, if only the thermal resistance of the sublayer $(0 < y+ < 30)$ is considered,

$$\frac{h}{\rho c u_R} = \sqrt{(c_f/2)}/11 \tag{10}$$

Heat-transfer coefficients evaluated in this way are indicated by curves b and c on Fig. 11, for which evaluation the friction coefficients were taken as

$$c_f = 0{\cdot}030\left(\frac{\nu}{u_R x}\right)^{1/5},$$

with x measured along the bottom surface and with the velocity ratio $u_R/u_1 = 0{\cdot}60$, which Fig. 9 indicates to be a suitable estimate for at least the initial portion of the reverse flow. The prediction of equation (10) is closest to the experimental results and, recalling that the experimental values shown on Fig. 11 should be increased by about ten per cent to place the heat-transfer coefficient on the basis of the difference between the wall temperature and the temperature of the reverse flow, the correspondence in magnitude is quite close, though the experimental coefficients diminish with distance more rapidly than do the predicted values. This suggests an effect of the actual decrease in velocity with distance but any more critical evaluations are inappropriate because the friction law is not established and, as noted before, attempts to infer the friction were not particularly successful. The correspondence between equation (11) and experiment does support the view that almost all the thermal resistance is very close to the wall because of the high eddy diffusivity in the main part of the reverse flow, and this view is supported by the few temperature traverses that were made.

Front face

Beyond the separation point the values of the heat-transfer coefficient measured on the centerline require additional interpretation because of the spanwise components of velocity which exist in that region. On the bottom surface, only two additional thermocouples were available in this region, both 1 in from the centerline. These did not indicate departures of more than ten per cent from the centerline values of the heat-transfer coefficient. To obtain additional information, the front face of the cavity was altered to provide for additional thermocouples and the results of such measurements are shown on Fig. 13 for the 4 and 8 in notches for high and low speeds. A more severe variation of the heat-transfer coefficient is indicated there and this is greatest toward the outer edges and increases near the bottom of the front face. The correlation with velocity is retained throughout for the 4 in notch, but for the 8 in notch it fails progressively as the bottom is approached.

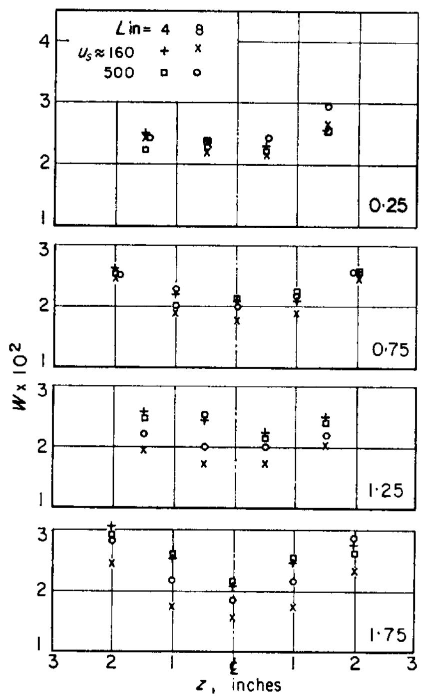

FIG. 13. Heat transfer on the front face.

The view is one facing the front face, with the abscissa the distance from the centerline. Results are shown for positions at 0·25, 0·75, 1·25 and 1·75 in below the top of the face.

The higher values of the heat-transfer coefficients near the outer edges of the front face agree with the results of the flow visualization, which indicated a spanwise flow inward to the centerline from the edges. It might be implied, from the more uniform behavior of the coefficients near the top of the notch, that the speed is more uniform there, but obviously no positive conclusion can be drawn. There remains the intriguing question as to the way in which the reverse flow, separated from the bottom surface, returns to the shear layer, either over or through the forward region of three-dimensional flow, and this may raise a question about the two-dimensionality assumed for the shear layer itself.

CONCLUSION

The presentation of results for the pressure coefficient, heat-transfer coefficient, and recovery factors for rectangular notch cavities of length from two to five times the depth, together with profiles of the mean speed in some of these cavities, has shown that at least a partial rationalization of the heat-transfer phenomena can be made for that region of the cavity in which the flow is essentially two dimensional. The shear layer, at least in its outer region, has been indicated to correspond with the shear layer typical of a free jet boundary, and a solution based on that view gives some of the features of the internal flow. This solution does not give the correct reverse velocities at the wall but if the magnitude thereof is appraised from the experimental results, the magnitude of the heat-transfer coefficient on the bottom of the cavity can be estimated from the usual specification for a flat plate, with, however, thermal resistance existing only in the flow sublayers.

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Résumé—Les coefficients de pression, les coefficients de convection, et les facteurs thermiques pariétaux ont été mesurés sur la surface de cavités peu profondes de forme rectangulaire, avec des rapports longueur sur profondeur de 2 à 5, pour un écoulement d'air sur la cavité allant de 49 à 180m/s. L'écoulement sur les surfaces de la cavité est bidimensionnel sur une partie de ces surfaces et, pour cette région, une rationalisation partielle du coefficient de convection est réalisée à partir de l'indication des profils de température et de vitesse moyenne obtenus en divers points en aval du décollement. L'importance de la couche libre de cisailleement au-dessus du sommet de la cavité est soulignée et on a montre que certaines de ses propriétés correspondent à celle de la limite d'un jet libre.

Zusammenfassung—An der Oberfläche rechteckiger Vertiefungen mit einem Verhältnis von Länge zu Tiefe von 2 zu 5 wurden für Luft Druckkoeffizienten, Wärmeübergangszahlen und Rückgewinnfaktoren gemessen bei Geschwindigkeiten von 49 m/s bis 180 m/s. Die Strömung an der Oberfläche der Vertiefung erweist sich zum Teil als zweidimensional und für diesen Bereich ermittelt man die Wärmeübergangszahl aus der Temperaturanzeige und den mittleren Geschwindigkeitsprofilen, die man an verschiedenen Stellen nach der Ablösung erhält. Die Wichtigkeit der freien Scherschicht über die ganze Oberseite der Vertiefung wird betont und es wird gezeigt, dass eine Anzahl ihrer Merkmale jenen an der Begrenzung eines Freistrahles entspricht.

Аннотация—Проведено измерение коэффициентов давления, теплообмена и восстановления на поверхности впадин прямоугольной формы с отношениями длины к глубине 2:5 при скорости движения воздуха над впадиной 160 ÷ 590 фут/сек. Покащано, что течение во впадине двумерно лишь над некоторой частью ограничивающих впадину поверхностей. Для этой области возможна частичная рационализация коэффициента теплообмена. Это подтверждается измерениями температуры и профилями средней скорости на различных расстояниях от места отрыва потока. Установлено наличие свободного пограничного слоя над верхней частью впадины и показано, что его поведение сходно с поведением на свободной границе струи.